Digital System Algebra

Algebra :-

It is a type of Mathematics in which Letter (overy) used to represent possible quantity

→ The purpose of algebra it is to Simplify arithmetical and logical expression.

Boolean Algebra :-

→ In 1854, George Boolean, an English Mathematician developed algebra for simplifying the representation and manipulation of propositional logic

→ It is generally used to solve logical operation

→ It analyze and solve Boolean expression

→ It didn't had any practical use since 1938, when Shannon used it first time in telephone switching circuit

→ It Shannon's work gave an idea that Boolean algebra could be idea that applied to computer electronics.

→ It is Most basic tool to anlyze analyze and design logic circuit.

* Boolean Variable :-

→ In algebra we take a variable as the one that can take different values at different instant of time

→ However a Boolean variable is a variable that is capable of taking only two values, states,.

→ these states/value are represented by on or off true or false].

* Boolean function (expression:—

→ In ordinary algebra we have the concept of expression or function similary in boolean algebra we have the concept of expression or function consist of Boolean variable -

Eg - $X \cdot (A \cdot B \cdot C \cdot D \cdot A + B)$

Here, A, B, C, D are Boolean variable.

Truth table

→ It is the pictorial representation. of a boolean expression containing Boolean variable for showing result of all possible :

→ It is combination of input.

→ the input boolean variable will have two states or 1 0, 1.

→ the combination of all such input value will result into an output value to be or 1

* Logic gate

→ Logic gates are the basic building blocks of any digital system,

→ It is an electronic circuit having one or More than one input and only one output

→ the relationship between the input and the output is based on.

Operators called logic operators.
Logic operators are :-

- OR Operators
- AND operators
- NOT Operators
- NOR operators
- Exclusive operators -OR
- exclusive NOR operators.

Logical or operator :-

we can operator two Booleon Variables using a logical or operator the result of an of on or operator is logic 1 if one the input value is 0 then out logic if all the value are 0 thus A and B will put out value is 0 Meaning it is repr- convey a logical esented by (+).

Boolean function :-

$X = A + B$

Truth-table

A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

Logical and operator :-

The result of an AND operator is logic 1 when all the input values are logic Boolean function

$$X = A \cdot B$$

Truth table :-

A	B	A·B
0	0	0
0	1	0
1	0	0
1	1	1

$$X = A \cdot B \cdot C$$

A	B	C	A·BC
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0
		0	0
		1	0

Logical Not gate:-

Not operator in Boolean algebra is used to describe the opposite or complimentary state/value of that variable:-

A Boolean variable is Not operation is.

If $A=1$, than $x==0$
If $A=0$. than $x=1$

It is denoted by a bar (-) and pronounced as complement of A

Boolean function

$$Y = \bar{A}$$

Truth table:-

A	$\bar{A}$
0	1
1	0

Logical NOR operator:-

Not operator in Boolean algebra is used to describe the opposite or complimentary state/value of that variable:-

A Boolean variable is Not operation.

NOR operator is logically opposite of or gate it gives output 1 only if all input are in a state,

Boolean function

A	B	0+1	$Y = \overline{A+B}$
0	0	0	1
0	1	0	1
1	0	1	0
1	1	1	0

Logic NAND operatoor :—

It is just opposite of AND gate. NAND gate gives output logic 1 when any one of variable value is logic 0.

$S = \overline{x+y+z}$ $\quad$ $S = \overline{x+y}$

Truth table :—

X	Y	Z	$S = \overline{x+y+z}$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

X	X	$S = XY$
0	0	1
0	1	1

1	0	1
1	1	0

Exclusive OR (or XOR) Operator :-

If both the value is same then its output is in 0 state otherwise output is in 1 state.

$Z = A \cdot B + A \cdot B$ or $Z = A + B$.

Truth table :-

A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	0

Que If two gates are connected with AND gate what will be the output Z for input A & B.

Truth table :-

A	B	A'	B'	A'B'
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	0

Q - Solve the following expression :-

i $x = A + B'C$,

I

A	B	C	B'	A+B	X
0	0	0	1	1	0
0	0	1	1	1	1
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	1	1	0
1	0	1	1	1	1
1	1	0	0	1	0
1	1	1	0	1	1

II $X = AC + A'B'C$

A	B	C	A'	B'	A·C	A'B'C'	X
0	0	0	1	1	0	0	0
0	0	1	1	1	0	1	1
0	1	0	1	0	0	0	0
0	1	1	1	0	0	1	1
1	0	0	0	1	1	0	0
1	0	1	0	1	1	1	1
1	1	0	0	0	1	0	0
1	1	1	0	0	1	1	1

Postulates of Booleon algebra :-

(i) Fundamental condition ove self-evident Proposition are called Postulate,

(ii) the Postulates of Booleon algebra

originates from 3 basic operation and or OR and NOT.

(iii) the Properties of these basic operation are called Postulates of Boolean algebra

(iv) In the other words the result of these operation are called Postulates of Boolean algebra,

(v) Postulates of Boolean algebra are:—

$$\boxed{\begin{aligned} 0\cdot0&=0\\ 0\cdot1&=0\\ 1\cdot0&=0\\ 1\cdot1&=0 \end{aligned}}$$ = derived from AND operator.

$$\boxed{\begin{aligned} 0+0&=0\\ 0+1&=1\\ 1+0&=1\\ 1+1&=0 \end{aligned}}$$ = derive from OR operation

$$\boxed{\begin{aligned} 0&=1\\ 1&=0 \end{aligned}}$$ = derive from Not operation

* <u>Demorgan's theorem:—</u>

theorem:— the complement of a Sum is equal to the Product of it's complement:-

Proof :-

Case 1 : 0.0

L.H.S = A+B = 0+0 = 0 = 1

R.H.S = A'.B' = 0 = 1.1 = 1

Case 2 :- 0.1

L.H.S = A+B = 0+1 = 1' = 0

R.H.S = A.B = 0.1 = 1.0 = 0

Case 3 :- 1.0

L.H.S = A+B = 1+0 = 1 = 0

R.H.S = A.B' = 1.0' = 0.1 = 0

Case 4 :- 1.1

L.H.S = A+B = 1+1 = 1 = 0

R.H.S = A'B = 1.1 = 0.0 = 0

L.H.S = R.H.S.

A+B = A'B Prove that.

A	B	A+B	A+B	A'	B'	A'B
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

From above table

L.H.S = R.H.S.

Theorem 2 :- The compliment of Product is equal to sum of it's compliment

$$A \cdot B = A + B.$$

A	B	A·B	A·B	A'	B'	A'+B'
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

From above table
L·H·S = R·H·S

Boolean theorem :-

1. 0·A = 0
2. A·0 = 0
3. 1·A = A
4. A·1 = A

Properties of AND operator.

5 0+A = A
6 A+0 = A
7 A+1 = 1
8 1+A = 1

Properties of OR operator.

9 A·A = A
10 A·A = 0
11 A+A = A
12 A+A = 1

Combining a Variable with itself are it's complement.

14 A+B = B = B+A
15 A·B = B·A

Commutative Law,

16 $A+(B+C) = (A+B)+C$
17 $A\cdot(B\cdot C) = (A\cdot B)\cdot C$ Associative Law.

18 $A\cdot(B\cdot C) = A\cdot B + A\cdot C$
19 $A+(B+C) = (A+B)\cdot(A+C)$
Distributive Law.

20. Demorgans, law.

21 $A+A'\cdot B = A+B$
22 $A+A\cdot B = A'+B$,

Q. Simplify the following operation:—

1. $xy'z' + xy'z'w + xz'$
$= xy'z'(1+w) + xz'$
$= xy'z' + xz'$
$= xz(y'+1)$ $(1+w=1)$
$= xz'$ $(y'+1=1)$

2. $x'y'z' + x'y'z + xy'z' + xyz'$
$= x'z'(y'+y) + xz(y'+y)$
$= x'z + xz'$
$= z(x'+x)$ $(x'+y=1)$
$= z$.

3) $z(y+z)(x+y+z)$

$= (zy+zz)(x+y+z)$

$= z(y+1)(xy+y+z)$

$= z(x+y+z)$

$= zx+zy+zz$

$= zx+z(y+1)$

$= zx+z$

$= z(x+1)$

$= z$ Ans

* Karnaugh ~~Law~~ Map

i) Karnaugh Map Method is a graphical technique For simplifying some Boolean expression

ii) it is a 2-D representation of a truth table.

iii) Karnaugh Map Provides a simpler Method for simplifying Boolean function

iv) the Map Method is ideally suitable for four are less variable

v) A Karnaugh Map is a diagram consisting of square, each Square of Map represents Mean term.

vi) Karnaugh Map for n variable is Made up of 2^n square each square designates a a Product term of a Boolean expression.

vii) Product term which are present expression is written in the corresponding Square.

Eg → $Y = AB' + AB$.

A'	A
	1
	1

In Karnaugh Map for 3 Variable the ordering of the Variable i.e. 00, 01, 11, 10 is binary code they do Not State binary code 00, 01, 10, 11 it is VEITCH diagrams while forming group of adjacent Square containing is the following consideration Must be kept in mind-

1. Every Square containing 1 Must be considered attest once.

2. A Square containing 1 can be included in as Many groups as desired A group Must be as Large as Possible.

3. The Number of Square in groups Must be equal to 2n Such as 2, 4, 8, 16 Not 3. 5. 6. 7

* UNIT-4

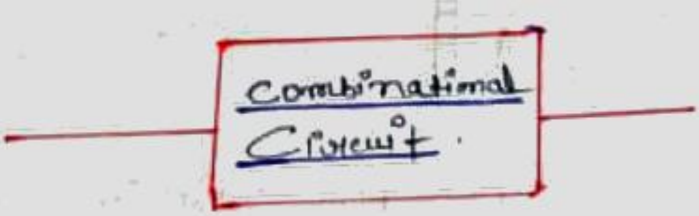
COMBINATONAL SWITCHING CIRCUITS

Introduction :- There are two types of logical circuit.

1. Combinational Circuit
2. Squential Circuit

1. Combinational circuit :-

(i) A logical device where output value at any given instant depends only upon the input value at the time



(ii) Combinational circuit is described as Boolean expression and truth table. Truth table list the each combinatonal of input value.

(iii) A combinational circuit consist of logical gate whose output at any time are determine by combining the value of input.

(iv) combinational circuit are realized with AND, OR and Not gates.

(v) Combinational

(v) combinational circuit are adder, Subtractor, encoder, Multiplier, divisor etc

(vi) four n input variable there are 2^n possible binary input combination $n=3, 2^3=8$

(vii) four each binary combination of the input variable, there are one possible binary value on which output.

2. Sequential Circuit :-

Sequential circuit consist of combinational circuit as well as Memory element use to store contain circuit state). In other word output depend upon the both current input value (which are kept in the storage element).

Combinational v/s Sequential circuit :—

1. combinational circuit is memory less thus, the output value depends upon on the current output value.

whether sequential circuit consist combinational circuit as well as memory element used to store certain circuit states.

ii. output element used to store certain output value depends on current output in combination circuit

whether in sequential circuit output value depends on both current and previous input value,

* Combinational Circuit design Procedure :-

i) State the Problem.
ii) find out the input and output variable
iii) use an appropriate coding to represent input/ output variables.
iv) Assigning a letter symbols to represent Of the Problem.
v) Obtain a truth table using the Statement Of the Problem.
vi) Represent each output variable in truth table.

* Integrated circuit :-

i) Informally a "chip" a conductor crystal Most of an silicon containing the electronic component four the digital gates

ii) Storage elements which are inter connected on the chip.

iii) The circuit is said to be integrated because

v) There are following Levels of I c.

a] SSI :- Small Scale Integration.
b] MSI :- Medium Scale integration [10 to 100 gates]
c] LSI :- Large Scale integration
d] VLSI :- Very Large Scale integration
e] ULSI :- Ultra Large Scale integrati-

Input unit		Output unit	
A	B	Sum(S)	carry (C)
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Sum (S) = $A'B + AB' = A + B$

Carry (C) = $A \cdot B$

Sum(S) $A'B + AB'$
$= A + B$
Carry(C), A, [illegible]

Full Adder :- it is combinational circuit which can add three binary bits.

Sum(S) = $A'B'C + A'BC' + AB'C' + ABC$

Carry(C) = $A'BC + AB'C + ABC' + ABC$

$= A'B'C + A'B'C' + AB'C + ABC$

$= A'(B'C + BC') + AB(B'C' + BC)$

$= A'(B+C) + A(B+C)$

$= A'(B+C) + A(B+C)$

Let, $B+C = Z$

$= AZ + AZ'$

$= a+z$

$= a+(B+C)$ [Place + Means exclusive symbol]

Q. $C = A'BC + AB'C + ABC' + ABC$

$= A'BC + AB'C + ABC' + ABC + ABC$

$= A'BC + ABC + AB'C + ABC + ABC' + ABC$

$= BC(A'+A) + AC(B'+B) + AB(C'+C)$

$= BC + AC + AB,$

A	B	C	Sum(S)	Carry(C)
0	0	0	0	0

End